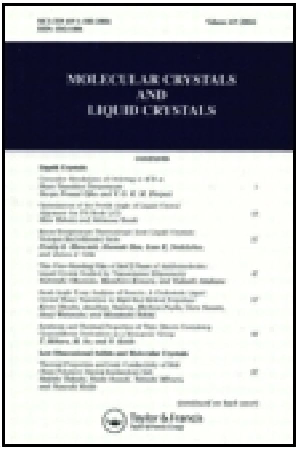

Molecular Crystals and Liquid Crystals

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl20

FLOW EFFECT IN DIRECTOR RELAXATION OF BOOKSHELF ALIGNED SMECTIC C LIQUID CRYSTALS

P. D. Woods [a] , I. W. Stewart [a] & N. J. Mottram [a]

[a] Department of Mathematics, University of Strathclyde, Livingstone Tower, 26 Richmond Street, Glasgow G1 1XH, United Kingdom

Version of record first published: 07 Jan 2010

Mol. Cryst. Liq. Cryst., Vol. 413, pp. 271/[2407]–278/[2414], 2004

ISSN: 1542-1406 print/1563-5287 online
DOI: 10.1080/15421400490437088

FLOW EFFECT IN DIRECTOR RELAXATION OF BOOKSHELF ALIGNED SMECTIC C LIQUID CRYSTALS

P. D. Woods, I. W. Stewart, and N. J. Mottram
Department of Mathematics, University of Strathclyde, Livingstone Tower, 26 Richmond Street, Glasgow G1 1XH, United Kingdom

We present a theoretical investigation of the director orientation and induced flow in a 'bookshelf' aligned cell of smectic C liquid crystal. The director is shown to relax to equilibrium and induce a flow when a high magnitude electric field is suddenly removed.

Keywords: flow in smectic C; relaxation in smectic C; smectic C liquid crystals

INTRODUCTION

We consider the flow induced after a high magnitude electric field is removed and the director structure is allowed to relax to equilibrium in planar aligned 'bookshelf' samples of non-chiral smectic C where the boundary conditions for the director on each bounding plate are different. Such a flow is therefore similar to the flow effect in nematic liquid crystals. The initial director configuration is taken to be that of a domain boundary where part of the layer has been switched such that the director lies on one side of the usual smectic cone in the bulk of the sample while near the boundaries it lies on different positions on the cone. The director structure is therefore an internal boundary layer or static solitary wave separating the switched and unswitched regions. In this geometry the nonlinear dynamic theory of smectic C liquid crystals introduced by Leslie, Stewart and Nakagawa [1] leads to three governing equations, one for the azimuthal angle of the director around the cone and two for the two components of the fluid velocity within the layer. Motivated by work on similar problems

Address correspondence to P. D. Woods, Department of Mathematics, University of Strathclyde, Livingstone Tower, 26 Richmond Street, Glasgow, Scotland G1 1XH, United Kingdom.

from nematic and smectic C liquid crystals [2,3], we analytically model the relaxation through approximating equations to these highly nonlinear equations and produce plots exhibiting the essential features. We find that the director rotation, upon relaxation to the equilibrium state, induces a flow at an angle (at a right-angle in the early stages of the dynamics) to the previously applied electric field.

Using the Leslie-Stewart-Nakagawa theory for smectic C liquid crystals we make two assumptions: that the initial smectic layer configuration remains intact upon the removal of the applied field, and that the tilt angle of the director with respect to the layer normal remains fixed.

Much interest has been given to smectic liquid crystals due to their faster switching compared to nematic displays. However, results on flow during switching in smectics rather than nematic liquid crystals are less understood. Here we present some preliminary results on the director orientation and flow due to director relaxation after an electric field is removed in smectic C liquid crystals. We examine the flow induced in a smectic C cell by rotation of the director around the smectic cone. In this study we have looked at the bookshelf geometry in which the smectic layers are perpendicular to the bounding plates (see Fig. 1), with strong anchoring at the surfaces. The liquid crystal is assumed to have a positive dielectric anisotropy, indicating that the director prefers to align with the electric field. We further suppose that the application of a strong electric field perpendicular to the plates (in the z direction) has distorted the director structure through the coupling of the director n and the electric field so that the c-director is mostly aligned parallel to the field (see Fig. 1(c)). We

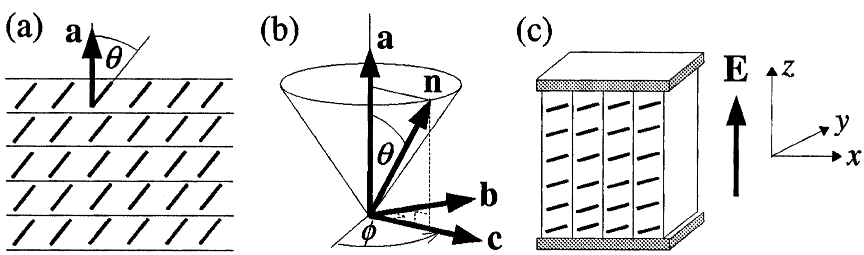

FIGURE 1 (a) The smectic C layer description. The short bold lines represent the director alignment. The director makes a fixed angle θ with respect to the unit layer normal **a**. (b) Description of the director **n** around the fictitious smectic cone. The c-director **c** is the unit orthogonal projection of **n** onto the smectic planes and is described by the orientation angle ϕ. The vector $\mathbf{b} = \mathbf{a} \times \mathbf{c}$ is introduced for convenience. (c) The 'bookshelf' geometry for smectic C and the coordinate system used in the text. The electric field **E** is removed at $t = 0$, initiating the relaxation of the director **n**.

then consider the relaxation that occurs when the field is suddenly removed.

When a high magnitude electric field is initially applied across the cell, the director attempts to fully align the sample at $\phi = \frac{\pi}{2}$ (see Fig. 1(c) and Eq. $(1)_2$ below), so that the c-director is parallel with the field direction, indicating that the director prefers to lie on one position of the smectic cone where **n** is most parallel to the field, except near boundary surfaces where surface treatment fixes the director to lie on other positions on the cone.

The set of boundary conditions we consider is anti-symmetric. In this case, the phase angle of the c-director is $\phi = 0$ at the lower plate and $\phi = \pi$ at the upper plate. The usual no-slip boundary condition implies that the flow velocity is zero at both plates.

THEORY

The smectic C state may be described by two unit vectors, the unit layer normal **a** perpendicular to the smectic layers and a unit vector **c** perpendicular to **a** describing the direction of tilt with respect to the layer normal (see Fig. 1(b)).

If we assume **a** is constant and has a component solely in the x-direction and that the flow has no component in the z-direction, then we can write

$$\mathbf{a} = (1, 0, 0), \mathbf{c} = (0, \cos\phi(z,t), \sin\phi(z,t)), \ \mathbf{v} = (u(z,t), v(z,t), 0), \qquad (1)$$

where it is assumed that ϕ, u and v depend only upon z and t.

The onset of relaxation of the director occurs when the applied field is suddenly removed. The layers are assumed to remain of constant thickness and the smectic tilt angle, θ, of the director with respect to the layer normal is assumed to remain constant. The continuum theory then delivers three equations for ϕ, u and v. The equation governing the azimuthal angle ϕ of the director is [3]

$$\begin{aligned} &K_2^c\left[\cos\phi\frac{\partial^2\phi}{\partial z^2} - \left(\frac{\partial\phi}{\partial z}\right)^2\sin\phi\right]\cos\phi \\ &\quad + K_3^c\left[\sin\phi\frac{\partial^2\phi}{\partial z^2} + \left(\frac{\partial\phi}{\partial z}\right)^2\cos\phi\right]\sin\phi - 2\lambda_5\frac{\partial\phi}{\partial t} \\ &\quad - (\tau_1+\tau_5)\frac{\partial u}{\partial z}\cos\phi - (\lambda_5 + \lambda_2\cos 2\phi)\frac{\partial v}{\partial z} = 0. \end{aligned} \qquad (2)$$

K_2^c and K_3^c are (positive) smectic elastic constants related to distortions of the c-director [3,4] and $\tau_1, \tau_5, \lambda_2$ and λ_5 are smectic viscosities [1], where it is known *a priori* that $\lambda_5 > 0$. The viscosity λ_5 is the rotational viscosity of the director around the surface of the smectic cone, somewhat analogous to the usual rotational viscosity of nematic liquid crystals. The equations governing the two components u and v of the fluid velocity within the smectic layers are [3]

$$\rho \frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\left[\xi_1(\phi)\frac{\partial u}{\partial z} + \xi_2(\phi)\cos\phi\frac{\partial v}{\partial z} + (\tau_1+\tau_5)\cos\phi\frac{\partial \phi}{\partial t}\right], \tag{3}$$

$$\rho \frac{\partial v}{\partial t} = \frac{\partial}{\partial z}\left[\xi_2(\phi)\cos\phi\frac{\partial u}{\partial z} + \xi_3(\phi)\frac{\partial v}{\partial z} + (\lambda_5+\lambda_2\cos 2\phi)\frac{\partial \phi}{\partial t}\right], \tag{4}$$

where ρ is the density of the liquid crystal and

$$\begin{aligned} 2\xi_1(\phi) &= \mu_0+\mu_2+2\lambda_1+\lambda_4 \\ &\qquad + (\mu_4+\mu_5-2\lambda_2+2\lambda_3+\lambda_5+\lambda_6)\sin^2\phi, \\ 2\xi_2(\phi) &= \kappa_1+\tau_1+\tau_2+\tau_5+2(\kappa_3+\tau_4)\sin^2\phi, \\ 2\xi_3(\phi) &= \mu_0+\mu_4+\lambda_5+2\mu_3\sin^2\phi\cos^2\phi+2\lambda_2\cos 2\phi, \end{aligned} \tag{5}$$

where the quantities μ_i, κ_i, τ_i and λ_i are further smectic viscosities. New normalised variables are introduced

$$\zeta = \frac{z}{d}, \quad k = \frac{K_3^c}{2\lambda_5}, \quad \tau = \frac{kt}{d^2}, \quad \widehat{u} = \frac{du}{k}, \quad \widehat{v} = \frac{dv}{k}, \tag{6}$$

where $2d$ is the cell depth in the z-direction.

As is common in the theory of nematics, we neglect the inertial terms appearing on the left-hand sides of Eqn. (3) as they are considered negligible in many general liquid crystal problems. On taking an initial director orientation of $\phi = \pi/2$, the flow Eqn. (3) may be approximated in the early stages of the dynamics by

$$0 = \xi_1\frac{\partial^2 \hat{u}}{\partial \zeta^2}, \tag{7}$$

$$0 = \xi_3\frac{\partial^2 \hat{v}}{\partial \zeta^2} + (\lambda_5-\lambda_2)\frac{\partial^2 \phi}{\partial \zeta \partial \tau}, \tag{8}$$

where we have now set $2\xi_3 = \mu_0+\mu_4+\lambda_5-2\lambda_2$. Equation (2) for the c-director orientation ϕ becomes, in a similar approximation,

$$\frac{\partial^2 \phi}{\partial \zeta^2} - \frac{\partial \phi}{\partial \tau} + \frac{(\lambda_2-\lambda_5)}{2\lambda_5}\frac{\partial \hat{v}}{\partial \zeta} = 0. \tag{9}$$

From the above equations it is apparent that $\hat{u}$ decouples from $\hat{v}$ and ϕ. Hence the only solution for $\hat{u}$ satisfying the no-slip boundary conditions is $\hat{u} \equiv 0$. If Eq. (9) is differentiated with respect to ζ it can be simplified. The resulting second derivative of $\hat{v}$ with respect to ζ can be expressed in terms of ϕ using Eq. (8). Hence it can be reduced to

$$\frac{\partial^3 \phi}{\partial \zeta^3} = \mu \frac{\partial^2 \phi}{\partial \zeta \partial \tau}, \tag{10}$$

where

$$\mu = 1 - \frac{(\lambda_5 - \lambda_2)^2}{2\xi_3 \lambda_5}. \tag{11}$$

This particular dimensionless constant μ, introduced in [3], is one possible measure of the reduction in the effective viscosity due to the presence of flow, analogous to that introduced into the well known theory of nematic liquid crystals.

For ease of computation, we can choose a suitably small fixed constant δ and approximate the initial condition on ϕ by the piecewise function:

$$\tilde{\phi}(\zeta, 0) = \begin{cases} \frac{\pi}{2\delta}(\zeta + 1) & \text{if} \quad -1 \le \zeta \le -1 + \delta, \\ \frac{\pi}{2} & \text{if} \quad -1 + \delta \le \zeta \le 1 - \delta, \\ \frac{\pi}{2\delta}(\zeta - 1) + \pi & \text{if} \quad 1 - \delta \le \zeta \le 1. \end{cases} \tag{12}$$

Using this, and by considering the Fourier series for $\phi(\zeta, 0)$ in ζ on [−1,1] and the expected long-time behaviour where $\phi \to \pi(1 + \zeta)/2$ as $\tau \to \infty$, we find that an appropriate ansatz for the solution ϕ at $\tau = 0$ is

$$\phi(\zeta, 0) = \frac{\pi}{2}(1 + \zeta) + \sum_{n=1}^{\infty} A_n \sin(n\pi\zeta), \tag{13}$$

$$A_n = -\frac{\sin((1 - \delta)n\pi)}{\delta \pi n^2}. \tag{14}$$

where the A_n represent the relevant Fourier coefficients, obtained by considering the initial switched state and the anticipated unswitched equilibrium state. To satisfy the expected long-time behaviour, $\phi(\zeta, \tau)$ has to decay to zero as $\tau \to \infty$. A solution that satisfies both this and the governing equations is

$$\phi(\zeta, \tau) = \frac{\pi}{2}(1 + \zeta) + \sum_{n=1}^{\infty} A_n \sin(n\pi\zeta) \exp(-\tau/\tau_n), \tag{15}$$

where $\tau_n = \mu / n^2 \pi^2$.

The solution for ϕ given by (15) can be substituted into Eq. (8) to obtain a solution for $\hat{v}$. Integrating twice with respect to ζ and then applying the appropriate no-slip boundary conditions reveals that we can set

$$\widehat{v} = \frac{\lambda_5 - \lambda_2}{\xi_3 \mu} \sum_{n=1}^{\infty} \pi n A_n [\cos(n\pi) - \cos(n\pi\zeta)] \exp(-\tau/\tau_n). \tag{16}$$

RESULTS

We start from the switched state and choose to take δ as 1/10 in the calculations involving the coefficients A_n defined in (14). Equations (15) and (16) are then simulated until the director has relaxed to the unswitched state. We choose to set

$$\lambda_2 = 0.0625\,\mathrm{Pa\,s}, \quad \lambda_5 = 0.03\,\mathrm{Pa\,s}, \quad 2\xi_3 = 0.0533\,\mathrm{Pa\,s}, \tag{17}$$

and take the sums in the above forms of the solutions to 2000 terms. The values for the viscosities λ_5 and $2\xi_3$ are those reported in the calculations

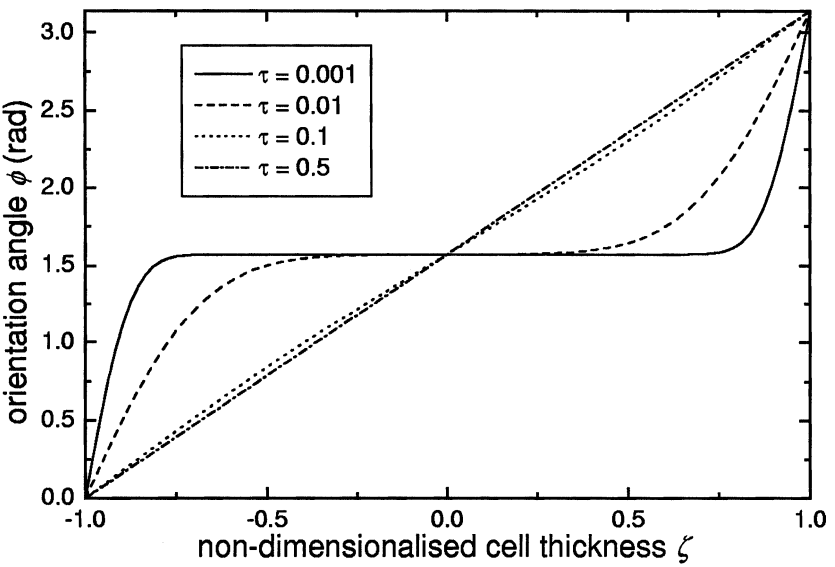

FIGURE 2 Relaxation of the director orientation angle $\phi(\zeta, \tau)$ across the sample for the indicated times.

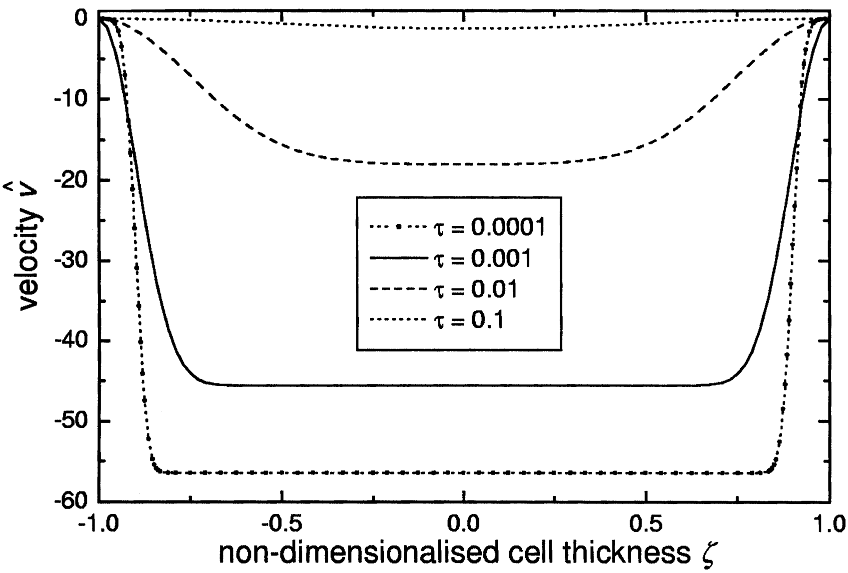

FIGURE 3 The fluid velocity $\hat{v}(\zeta, \tau)$ in the y-direction across the sample.

of Gill and Leslie [5], based upon the experimental work by Galerne, Martinand, Durand and Veyssie [6] on the SmC liquid crystal DOBPC at 103°C. The value for λ_2 is consistent with the calculation in [5] where it was shown that $|\lambda_5 - \lambda_2| = 0.0325\,\mathrm{Pa\,s}$. (Note that there is a minor miscalculation in [5].) These values give $\mu \doteqdot 0.34$.

Figure 2 shows that the orientation angle $\phi(\zeta, \tau)$ for the c-director decays smoothly back to its unswitched state after the field is removed. Figure 3 shows the fluid velocity $\hat{v}(\zeta, \tau)$. It is evident that the flow close to both boundaries in the initial and final stages of the relaxation is relatively small compared to that in the centre of the sample. In regions near the boundaries the flow initially increases before decaying to its final vanishing state.

CONCLUSIONS

The flow profile presented here is in contrast to the results contained in [7] for the corresponding planar aligned (homeotropic) smectic layers case. Further numerical work on the full nonlinear dynamic Eqs. (2)–(4) is forthcoming and seems to suggest that the flow component u, after the early stages of relaxation, also begins to be influential.

REFERENCES

[1] Leslie, F. M., Stewart, I. W., & Nakagawa, M. (1991). A continuum theory for smectic C liquid crystals. *Mol. Cryst. Liq. Cryst.*, *198*, 443–454.

[2] Clark, M. G. & Leslie, F. M. (1978). A calculation of orientational relaxation in nematic liquid crystals. *Proc. R. Soc. Lond. A*, *361*, 463–485.

[3] Blake, G. I. & Leslie, F. M. (1998). A backflow effect in smectic C liquid crystals. *Liq. Cryst.*, *25*, 319–327.

[4] Leslie, F. M., Stewart, I. W., Carlsson, T., & Nakagawa, M. (1991). Equivalent smectic C liquid crystal energies. *Continuum Mech. Thermodyn.*, *3*, 237–250.

[5] Gill, S. P. A. & Leslie, F. M. (1993). Some topics from continuum theory for smectic liquid crystals. *Ferroelectrics*, *148*, 11–24.

[6] Galerne, Y., Martinand, J. L., Durand, G., & Veyssie, M. (1972). Quasielectric Rayleigh scattering in a smectic C liquid crystal. *Phys. Rev. Lett.*, *29*, 562–564.

[7] Woods, P. D., Mottram, N. J., & Stewart, I. W. (2001). To appear in Ferroelectrics. Proceedings of the 8th International Conference on Ferroelectric Liquid Crystals, August, Washington D.C., USA.